MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2019

08-04-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above..

PART-A-CHEMISTRY

1. In the following compounds, the decreasing order of basic strength will be:

(1) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$ (2*) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$

(3) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$ (4) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$

Sol. Basic strength order

$(CH_3CH_2)_2NH > CH_3CH_2NH_2 > NH_3$

More the number of +I groups, higher is the basic strength.

2. For the reaction 2A + B $\rightarrow$ C, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is :

[A](molL^{-1})	[B](molL^{-1})	InitialRate (mol$L^{-1}s^{-1}$)
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

(1) Rate = k [A] [B] (2) Rate = k $[A]^2$ [B] (3*) Rate = k [A] $[B]^2$ (4) Rate = k $[A]^2$ $[B]^2$

Sol. $r = K[A]^x [B]^y$

$0.045 = K(0.05)^x (0.05)^y$ . . . . (1)

$0.090 = K(0.10)^x (0.05)^y$ . . . . (2)

$0.72 = K(0.20)^x (0.10)^y$ . . . . (3)

Dividing (1) by (2) we get

$$\frac{0.045}{0.090} = \left(\frac{0.05}{0.10}\right)^x \Rightarrow x = 1$$

Dividing (2) by (3)

$$\frac{0.090}{0.720} = \left(\frac{0.10}{0.20}\right)^x \left(\frac{0.05}{0.10}\right) \Rightarrow y = 2$$

3. The quantum number of four electrons are given below:

I. $n = 4, \ell = 2, m_\ell = -2, m_s = -\frac{1}{2}$

II. $n = 3, \ell = 2, m_\ell = 1, m_s = +\frac{1}{2}$

III. $n = 4, \ell = 1, m_\ell = 0, m_s = +\frac{1}{2}$

IV. $n = 3, \ell = 1, m_\ell = 1, m_s = -\frac{1}{2}$

The correct order of their increasing energies will be

(1) IV < III < II < I (2) I < III < II < IV (3*) IV < II < III < I (4) I < II < III < IV

Sol. According to Aufbau principle, the energy sequence is 3p < 3d < 4p < 4d

4. The lanthanide ion that would show colour is

(1*) Sm^{3+} (2) Lu^{3+} (3) La^{3+} (4) Gd^{3+}

Sol. $Sm^{3+}(4f^5)$ = yellow colour, other ions have stable electron configurations with half filled or full-filled electron configuration.

5. For silver, C_p $(JK^{-1}\ mol^{-1})$ = 23 + 0.01 T. If the temperature (T) of 3 moles of silver is raised from 300K to 1000 K at 1 atm pressure, the value of ΔH will be close to:

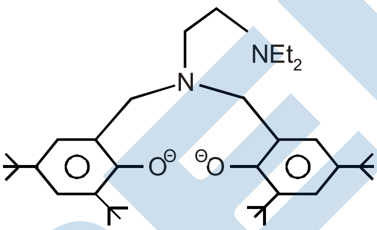
(1) 16 kJ (2) 13 kJ (3) 21 kJ (4*) 62 kJ

Sol. $\Delta H = n\int_{T_1}^{T_2} C_{p.m}dT = 3\times\int_{300}^{1000}(23+0.01T)dT$

$= 3[23(1000-300)] + \frac{0.01}{2}[(1000)^2-(300)^2]$

= 61950 J ≈ 62 KJ

6. The following ligand is :



(1) hexadentate (2) tridentate (3*) tetradentate (4) bidentate

Sol. n-factors of $KMnO_4$ = 5, n-factor of $FeSO_4$ = 1

n-factors of FeC_2O_4 = 3, $Fe_2(SO_4)_3$ does not react

n-factors of $Fe_2(C_2O_4)_3$ = 6,

$n_{eq}\ KMnO_4 = n_{eq}[FeC_2O_4 + Fe_2(C_2O_4)_3 + FeSO_4]$

or, x × 5 = 1 × 3 + 1 × 6 + 1 × 1

x = 2

7. If solubility product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S, then which of the following relation between S and K_{sp} is correct?

(1) $S = \left(\frac{K_{sp}}{216}\right)^{1/7}$ (2*) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$ (3) $S = \left(\frac{K_{sp}}{144}\right)^{1/6}$ (4) $S = \left(\frac{K_{sp}}{929}\right)^{1/9}$

Sol. $Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{4+}(aq.) + 4PO_4^{3-}(aq.)$

3s M 4S M

$K_{sp} = [Zr^{4+}]^3 [PO_4^{3-}]^4 = (3S)^3.(4S)^4 = 6912\ S^7$

$\therefore S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

8. The correct order of the spin-only magnetic moment of metal ions in the following low spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$, and $[Cr(NH_3)_6]^{2+}$ is

(1*) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

(2) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

(3) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$

(4) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$

Sol. Since CN^- and NH^3 are strong field ligands, low spin complexes are formed.

Complex	Configuration	No. of unpaired electrons
$[V(CN_6)]^{4-}$	$t^3_{2g}\ e^0_g$	3
$[Cr(NH_3)_6]^{2+}$	$t^4_{2g}e^0_g$	2
$[Ru(NH_3)_6]^{3+}$	$t^5_{2g}e^0_g$	1
$[Fe(CN)_6]^{14-}$	$t^6_{2g}e^0_g$	0

Magnetic moment is directly proportional to number of unpaired electrons..

9. The correct order of hydration enthalpies of alkali metal ions is

(1) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

(2) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$

(3) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$

(4*) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

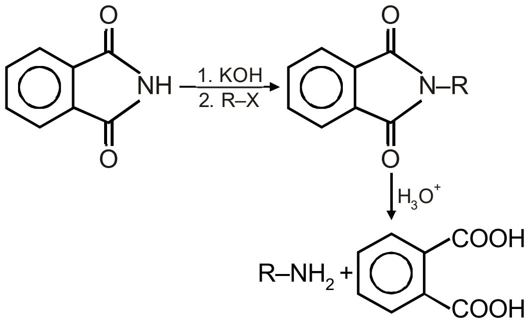
Sol. Hydration enthalpy depends upon ionic potential (charge/size). As ionic potential increases hydration enthalpy increases.

$\Delta_{hyd}H^0 \alpha \frac{q}{r}$

10. Which of the following amines can be prepared by Gabriel phthalimide reaction?

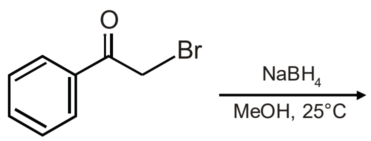
(1) t-butylamine (2*) n-butylamine (3) neo-pentylamine (4) triethylamine

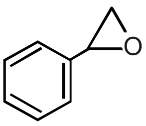
Sol. Gabriel phthalimide synthesis :

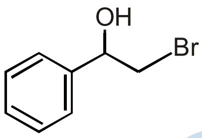


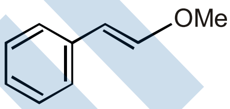
For branched chain RX, elimination reaction takes place.

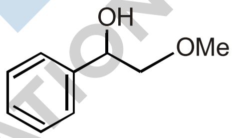
11. The major product of the following reaction is:

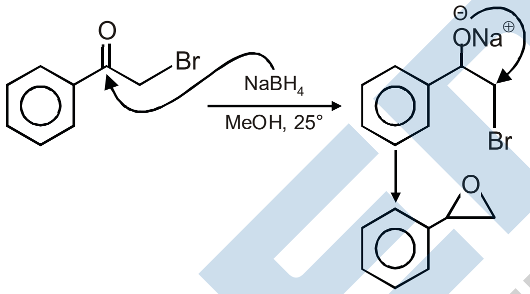


(1*) 

(2) 

(3) 

(4) 

Sol. 

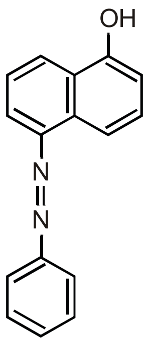
12. Given that; $E^{\circ}_{O_2/H_2O} = +1.23\text{ V}; E^{\circ}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05\text{ V}; E^{\circ}_{Br_2/Br^-} = +1.09\text{ V}; E^{\circ}_{Au^{3+}/Au} = +1.4\text{ V}$

The strongest oxidizing agent is:

(1*) $S_2O_8^{2-}$ (2) O_2 (3) Br_2 (4) Au^{3+}

sol. For strongest oxidising agent, standard reduction potential should be highest. Peroxy oxygen (–O – O–) is reduced to oxide (O^{2-}) in the change

13. Coupling of benzene diazonium chloride with 1–naphthol in alkaline medium will give:

(1)


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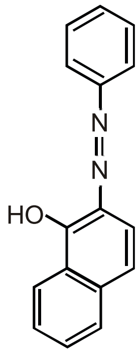

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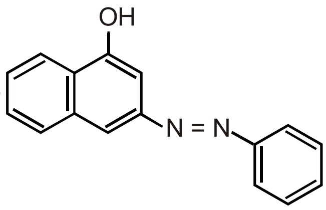

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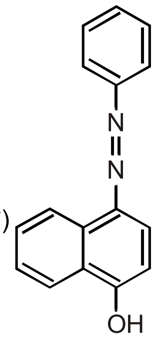
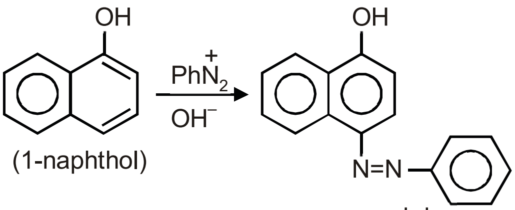
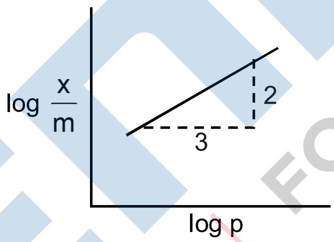


Sol.


(1-naphthol)
orange red dye

14. Adsorption of a gas follows Freundlich adsorption isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of log $\frac{x}{m}$ versus log p is shown in the given graph. $\frac{x}{m}$ is proportional to



(1) p^3 (2) p^2 (3*) $p^{2/3}$ (4) $p^{3/2}$

Sol. $\frac{x}{m} = Kp^{1/n}$

Taking log from both sides

$\therefore \log\frac{x}{m} = \log K + \frac{1}{n}\log P$

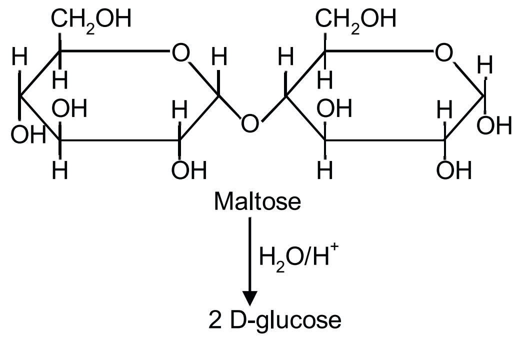
$\text{slope} = \frac{1}{n} = \frac{2}{3}$

$\therefore \frac{x}{m} = Kp^{2/3}$

15. Maltose on treatment with dilute HCl gives:

(1) D–Galactose
(2) D–Fructose
(3) D-Glucose and D–Fructose
(4*) D-Glucose

Sol.



16. In order to oxidise a mixture of one mole of each of FeC_2O_4, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is

(1) 1 (2) 3 (3*) 2 (4) 1.5

Sol. n-factors of $KMnO_4$ = 5, n-factor of $FeSO_4$ = 1

n-factors of FeC_2O_4 = 3, $Fe_2(SO4)_3$ does not react

n-factors of $Fe_2(C_2O_4)_3$ = 6,

$n_{eq}\ KMnO_4 = n_{eq}[FeC_2O_4 + Fe_2(C_2O_4)_3 + FeSO_4]$

or, x × 5 = 1 × 3 + 1 × 6 + 1 × 1

x = 2

17. 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is

(Molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 g mol^{-1})

(1*) 10,000 ppm (2) 1,000 ppm (3) 100 ppm (4) 5,000 ppm

Sol. $n_{eq.}\ CaCO_3 = n_{eq}Ca(HCO_3)_2 + n_{eq}Mg(HCO_3)_2$ (neq = Number of equivalent)

or, $\frac{W}{100} \times 2 = \frac{0.81}{162} \times 2 + \frac{0.73}{146} \times 2$

$\therefore$ w = 1.0

Volume of water = 100 mL

Mass of water = 100 g

$\therefore \text{Hardness} = \frac{1.0}{100} \times 10^6 = 10000 \text{ ppm}$

18. **Assertion:** Ozone is destroyed by CFCs in the upper stratosphere.

Reason: Ozone holes increase the amount of UV radiation reaching the earth.

(1) Assertion and reason are both correct, and the reason is the correct explanation for the assertion.

(2) Assertion is false, but the reason is correct.

(3) Assertion and reason are incorrect.

(4*) Assertion and reason are correct, but the reason is not the explanation for the assertion.

Sol. The upper stratosphere consists of ozone (O_3), which protect us from harmful ultraviolet (UV) radiations coming from sun. The layer get depleted by CFC's

19. With respect to an ore, Ellingham diagram helps to predict the feasibility of its

(1) Zone refining (2*) Thermal reduction (3) Electrolysis (4) Vapour phase refining

Sol. Ellingham diagram which are the curves of the graph between ΔG and T helps in predicting the feasibility of thermal reduction of ores.

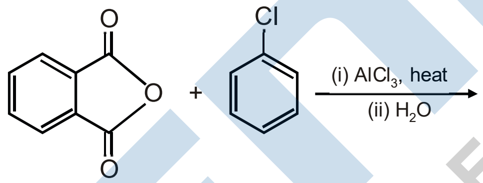
20. Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively:

(1*) B_2O_3 and H_3BO_3 (2) H_3BO_3 and B_2O_3 (3) B_2O_3 and $[BH_4]^-$ (4) HBO_2 and H_3BO_3

Sol. $B_2H_6 + 3H_2O \longrightarrow 2H_3BO_3 + 3H_2$

$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

21. The major product of the following reaction is:



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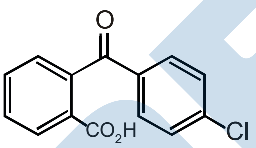

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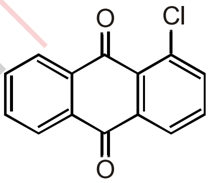

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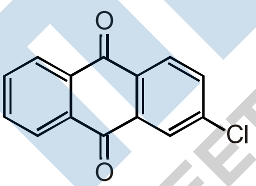

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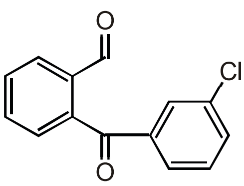
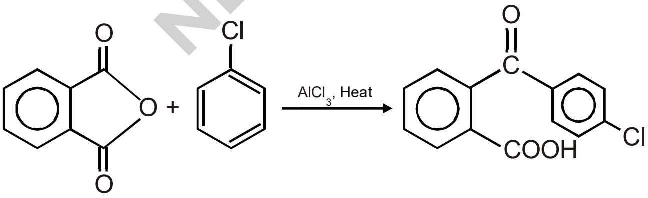
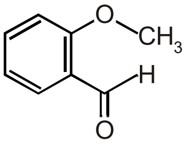
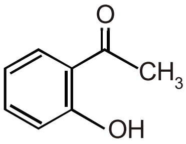
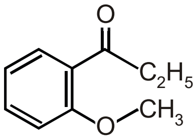
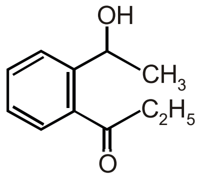
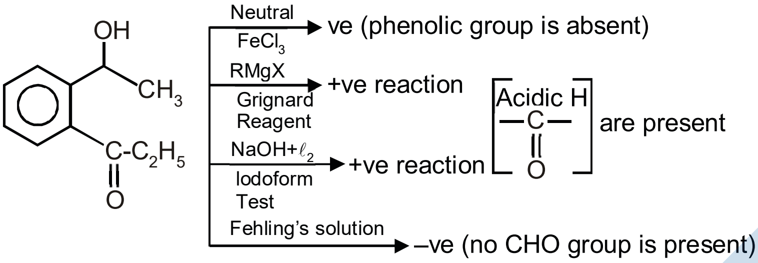


Sol.


Fridel-craft acylation. –Cl group is an ortho & para directing

22. An organic compound neither reacts with neutral ferric chloride solution nor with Fehling solution. It however, reacts with Grignard reagent and gives positive iodoform test. The compound is :

(1) 
(2) 
(3) 
(4*) 

Sol. 

23. The vapour pressures of pure liquids A and B are 400 and 600 mm Hg, respectively at 298 K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are:

(1) 500 mm Hg, 0.5, 0.5
(2*) 500 mm Hg, 0.4, 0.6
(3) 450 mm Hg, 0.5, 0.5
(4) 450 mm Hg, 0.4, 0.6

Sol. $P_{total} = X_A P_A^0 + X_B P_B^0 = 0.5 \times 400 \times 0.5 \times 600 = 500$ mmHg

Now, mole fraction of A in vapour

$Y_A = \frac{P_A}{P_{total}} = \frac{0.5 \times 400}{500} = 0.4$ and mole fraction of B in vapour

$Y_B = 1 - 0.4 = 0.6$

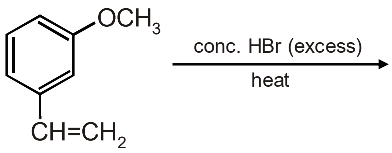
24. The size of the iso-electronic species Cl^-, Ar and Ca^{2+} is affected by

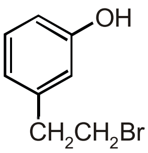
(1) azimuthal quantum number of valence shell
(2) principal quantum number of valence shell
(3*) nuclear charge
(4) electron-electron interaction in the outer orbital

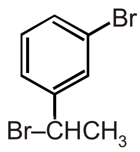
Sol. For isoelectronic species the size is compared by nuclear charge.

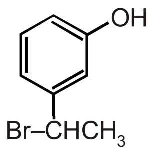
$$\text{Size} \propto \frac{1}{Z(\text{Nuclear Charge})}$$

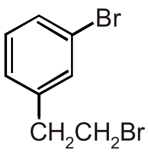
25. The major product of the following reaction is



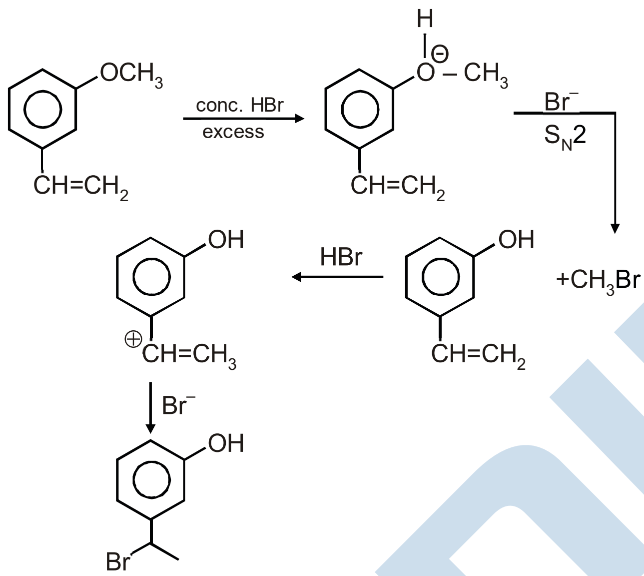
(1) 

(2) 

(3*) 

(4) 

Sol.



26. Which is wrong with respect to our responsibility as a human being to protect our environment?

(1*) Using plastic bags
(2) Restricting the use of vehicles
(3) Avoiding the use of flood lighted facilities
(4) Setting up compost tin in gardens

Sol. Plastic are non-biodegradable.

27. Element 'B' forms ccp structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is:

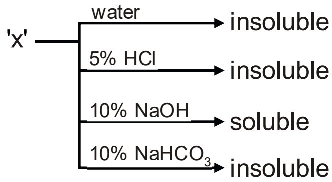
(1) A_2B_2O
(2) A_2BO_4
(3*) AB_2O_4
(4) A_4B_2O

Sol. For cubic unit cell, only FCC has octahedral and tetrahedral voids.

$$Z_B = 4,\ Z_A = 4 \times \frac{1}{4} = 2, Z_O = 8$$

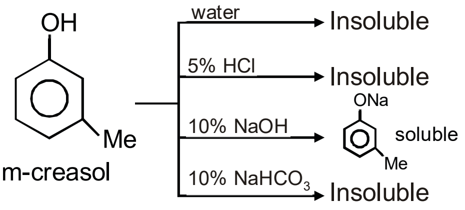
Formula = $A_2B_2O_8$ = AB_2O_4

28. An organic compound 'X' showing the following solubility profile is:



(1*) m-cresol (2) Benzamide (3) o-Toluidine (4) Oleic acid

Sol.



m-creasol

Oleic acid is also soluble in $NaHCO_3$
o-toluidine is not soluble in NaOH as well as $NaHCO_3$
Benzamide is also not soluble in NaOH & $NaHCO_3$
$\therefore$ m-cresol is the right answer.

29. The IUPAC name of the following compound is:

$$H_3C-\underset{|}{\overset{CH_3}{CH}}-\overset{OH}{\underset{}{CH}}-CH_2-COOH$$

(1) 4, 4–Dimethyl–3–hydroxybutanoic acid (2) 2–Methyl–3–hydroxypentan–5–oic acid

(3*) 3–Hydroxy–4–methylpentanoic acid (4) 4–Methyl–3–hydroxypentanoic acid

Sol. The priority of COOH is higher that OH.

$\therefore$ COOH is the functional group.

$$\underset{5}{CH_3}-\underset{4}{\overset{CH_3}{CH}}-\overset{3}{\underset{OH}{CH}}-\overset{2}{CH_2}-\overset{1}{COOH}$$

3-Hydorxy-4-methylpentanoic acid

30. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas? (Assume non expansion work is zero)

(1) Isochoric process : $\Delta U = q$ (2) Isothermal process : $q = -w$

(3*) Adiabatic process : $\Delta U = -w$ (4) Cyclic process : $q = -w$

Sol. According to the first law of thermodynamics $q = \Delta U - w$

For cyclic process : $\Delta U = 0 \Rightarrow q = -w$

For isothermal process : $\Delta U = 0 \Rightarrow q = -w$

For adiabatic process : $q = 0 \Rightarrow \Delta U = W$

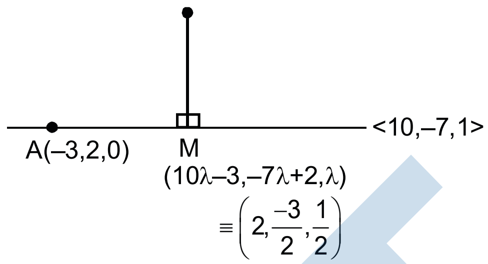
For isochoric process : $q = 0 \Rightarrow \Delta U = q$

PART-B-MATHEMATICS

31. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is

(1*) greater than 3 but less than 4
(2) greater than 2 but less than 3
(3) less than 2
(4) greater than 4

Sol. Now, $\overrightarrow{MP}.\left(10\hat{i}-\hat{i}\hat{j}+\hat{k}\right)=0$



$\Rightarrow \lambda=\frac{1}{2}$

∴ Length of perpendicular

$(=PM)=\sqrt{0+\frac{1}{4}+\frac{49}{4}}$

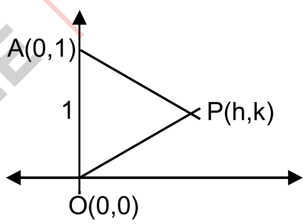
$=\sqrt{\frac{50}{4}}=\sqrt{\frac{25}{4}}=\frac{5}{\sqrt{2}},$

Which is greater then 3 but less than 4.

32. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is

(1) $8x^2 + 9y^2 - 9y = 18$
(2) $8x^2 - 9y^2 + 9y = 18$
(3) $9x^2 - 8y^2 + 8y = 16$
(4*) $9x^2 + 8y^2 - 8y = 16$

Sol. AP + OP + AO = 4



$\sqrt{h^2+\left(k-1^2\right)}+\sqrt{h^2+k^2}+1=4$

$\sqrt{h^2+\left(k-1^2\right)}+\sqrt{h^2+k^2}=3$

$-2k - 8 = -6\ \sqrt{h^2+k^2}$

$k + 4 = 3\ \sqrt{h^2+k^2}$

$k^2 + 16 + 8k = 9(h2 + k^2)$

$9h^2 + 8k^2 - 8k - 16 = 0$

Locus of P is $9x^2 + 8y^2 - 8y - 16 = 0$

33. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n=1$ is

(1*) 4
(2) 5
(3) 2
(4) 3

Sol. $(X - 1)^2 + 1 = 0 \Rightarrow X = 1 + I,\ 1 - i$

$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$

$\therefore$ n (least natural number) = 4

34. The mean and variance of seven observations are 8 and 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14 then the product of the remaining two observations is

(1) 40 (2*) 48 (3) 49 (4) 45

Sol. Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56$..............(1)

Also $\sigma^2 = 16$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$

$\Rightarrow \left(\sum_{i=1}^{7} x_i^2\right) = 560.$ ....(2)

Now, $x_1 = 2$, $x_2 = 4$, $x_3 = 10$, $x_4 = 12$, $x_5 = 14$

$\Rightarrow x_6 + x_7 = 14$, (from (1)) and $x^2_6 + x^2_7 = 100$ (from (2))

$\therefore x^2_6 + x^2_7 = (x_6 + x_7)^2 - 2x_6x_7 \Rightarrow c_6x_7 = 48$

35. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is

(1) 2 (2) –1 (3*) $\frac{1}{2}$ (4) 0

Sol. For non – trivial solution

D = 0

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0 \Rightarrow 2c^2 + 3c^2 - 1 = 0$$

$\Rightarrow (c + 1)^2 (2c - 1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

36. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then

(1*) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$
(2) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$
(3) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$
(4) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$

Sol. $f(x) = 9x^4 + 12x^3 + 36x2 + 25$

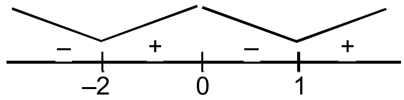
$f(x) + 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x - 1)(x + 2)$

Point of minima = $\{-2, 1\} = S_1$

Point of maxima = $\{0\} = S_2$



37. The area (in sq. units) of the region $A = \{(x, y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is

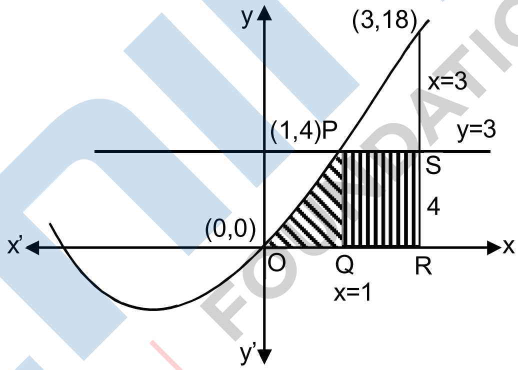
(1*) $\frac{59}{6}$
(2) $\frac{53}{6}$
(3) $\frac{26}{3}$
(4) 8

Sol. Required Area

$= \int_0^1 (x^2 + 3x)dx + \text{Area of}$

Rectangle PQRS

$= \frac{11}{6} + 8 = \frac{59}{6}$



38. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is

(1) $\frac{7}{8}$
(2) $\frac{11}{4\sqrt{2}}$
(3*) $\frac{7}{4\sqrt{2}}$
(4) 2

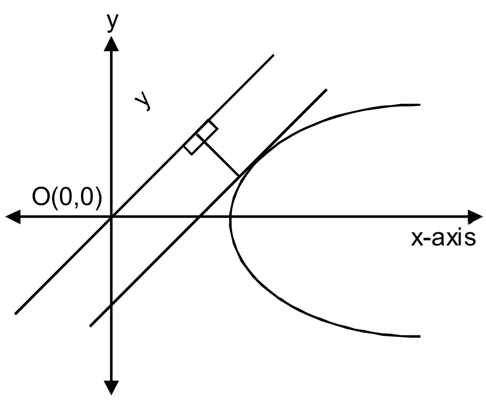
Sol. We have

$2y\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right]_{P(2+t^2,t)} = \frac{1}{2t} = 1$

$\Rightarrow t = \frac{1}{2}$

So, shortest distance

$= \frac{\left|\frac{9}{4}, \frac{1}{2}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$



39. If f (x) = $\log_e\left(\frac{1-x}{1+x}\right)$, | x | < 1, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

(1) 2f (x^2) (2) (f (x))^2 (3) –2f (x) (4*) 2f (x)

Sol. $f(x)=\log_e\left(\frac{1-x}{1+x}\right),|x|<1$

$$f\left(\frac{2x}{1+x^2}\right)=\ell n\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$=\ell n\left(\frac{(x-1)^2}{(x+1)^2}\right)=2\ell n\left|\frac{1-x}{1+x}\right|=2f(x)$$

40. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is

(1) $\sqrt{6}$ (2*) $\sqrt{\frac{3}{2}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $3\sqrt{6}$

Sol. Vector perpendicular to plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ is parallel to

Vector

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$$

∴ Required magnitude of projection

$$=\frac{\left|\left(2\hat{i}+3\hat{j}+\hat{k}\right).\left(\hat{i}-2\hat{j}+\hat{k}\right)\right|}{\left|\hat{i}-2\hat{j}+\hat{k}\right|}$$

$$=\frac{|2-6+1|}{\left|\sqrt{6}\right|}=\frac{3}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

41. The sum of the series 2 · $^{20}C_0$ + 5 · $^{20}C_1$ + 8 · $^{20}C_2$ + 11 · $^{20}C_3$ + ... + 62 · $^{20}C_{20}$ is equal to

(1*) 2^{25} (2) 2^{26} (3) 2^{24} (4) 2^{23}

Sol. 2. $^{20}C_0$ + 5. $^{20}C_1$ + 8.|$^{20}C_2$ + 11. $^{20}C_3$ + ………+ 62. $^{20}C_{20}$2

$$=\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r.\,{}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r\left(\frac{20}{r}\right){}^{19}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$= 60.2^{19} + 2.2^{20} = 2^{25}$

42. The sum of squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the q lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is

(1) 160 (2*) 210 (3) 320 (4) 105

Sol. For non – trivial solution

D = 0

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0 \Rightarrow 2c^3 + 3c^2 - 1 = 0$$

$\Rightarrow (c + 1)^2 (2c - 1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

43. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right), \beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to

(1) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (3*) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{14}\right)$

Sol. $\cos\alpha = \frac{3}{5}, \tan\beta = \frac{1}{3}$

$\Rightarrow \tan\alpha = \frac{4}{3}$

$$\Rightarrow \tan(\alpha - \beta) = \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \cdot \frac{1}{3}} = \frac{9}{13}$$

$\Rightarrow \alpha - \beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

44. The sum of the co-efficients of all even degree terms in x in the expansion of

$\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$, $(x > 1)$ is equal to

(1) 32 (2*) 24 (3) 26 (4) 29

Sol. $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$= 2\,[{}^6C_0x^6 + {}^6C_2x^4(x^3 - 1) + {}^6C_4x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3]$

$= [{}^6C_0x^6 + {}^6C_2x^7 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^5 + (x^9 - 1 - 3x^6 + 3x^3]$

$\Rightarrow$ Sum of coefficient of even powers of x

= 2[1 – 15 + 15 + 15 – 1 – 3]

45. Let A = $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, $(\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is

(1) $\frac{\pi}{16}$ (2) $\frac{\pi}{32}$ (3*) $\frac{\pi}{64}$ (4) 0

Sol. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{bmatrix}$

Similarly $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow$ $\cos 32\alpha = 0$ and $\sin 32\alpha = 1$

$\Rightarrow 32\alpha = (4n+1)\frac{\pi}{2}, n \in \ell$

$\alpha = (4n-1)\frac{\pi}{65}, n \in \ell$

$\alpha = \frac{\pi}{64}$ for $n = 0$

46. Let y = y(x) be the solution of the differential equation, $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}\, y(1) = \frac{\pi}{32}$, then the value of 'a' is

(1) 1 (2*) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Sol. $\frac{dy}{dx} + \left(\frac{2x}{x^2 - 1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. $= e^{\ell n(x^2+1)} = (x^2 + 1)$

So, general solution is $y.(x^2 + 1) = \tan^{-1} x + c$

As $y(0) = 0 \quad \Rightarrow \quad c = 0$

$\therefore \quad y(x) = \frac{\tan^{-1} x}{x^2 + 1}$

$As, \sqrt{a}, y(1) = \frac{\pi}{32}$

$\Rightarrow \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$

47. Let f : [0, 2] → R be a twice differentiable function such that f "(x) > 0, for all x ∈ (0, 2).

If ϕ (x) = f (x) + f (2 – x), then ϕ is

(1) decreasing on (0, 2)

(2) increasing on (0, 2)

(3*) decreasing on (0, 1) and increasing on (1, 2)

(4) increasing on (0, 1) and decreasing on (1, 2)

Sol. $\phi(x) = f(x) + f(2 - x)$

$\phi'(x) = f(x) - f'(2 - x)$.............(1)

Since f"(x) > 0

$\Rightarrow$ f(x) is increasing $\forall$ x ∈ (0,2)

Case – I : When x > 2 – x $\Rightarrow$ x > 1

$\Rightarrow \phi'(x) > 0 \forall x \in (1,2)$

$\therefore \phi(x)$ is increasing on (1, 2)

Case – II : When x > 2 – x $\Rightarrow$ x < 1

$\Rightarrow \phi'(x) < 0 \ \forall x \in (0,1)$

$\therefore \ \phi(x)$ is decreasing on (0, 1)

48. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$, $x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to

(1) $\frac{\pi}{6} - x$ (2) $2x - \frac{\pi}{3}$ (3) $\frac{\pi}{3} - x$ (4*) $x - \frac{\pi}{6}$

Sol. Consider $\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\sin x}\right)$

$= \cot^{-1}\left(\frac{\sin\left(x + \frac{\pi}{3}\right)}{\cos\left(x + \frac{\pi}{3}\right)}\right)$

$$= \cot^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right) = \frac{\pi}{2} - \tan^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)$$

$$\begin{cases} \frac{\pi}{2} - \left(x+\frac{\pi}{3}\right) = \left(\frac{\pi}{6}-x\right);\ 0 < x < \frac{\pi}{6} \\ \frac{\pi}{2} - \left(\left(x-\frac{\pi}{3}\right)-\pi\right) = \left(\frac{7\pi}{6}-x\right);\ \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\therefore\quad 2y = \begin{cases} \left(\frac{\pi}{6}-x\right)^2;\ 0 < x < \frac{\pi}{6} \\ \left(\frac{7\pi}{6}-x\right)^2;\ \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\therefore\quad 2\frac{dy}{dx} = \begin{cases} 2\left(\frac{\pi}{6}-x\right).(-1);\ 0 < x < \frac{\pi}{6} \\ 2\left(\frac{7\pi}{6}-x\right).(-1);\ \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

49. A point on the straight line, 3x + 5y = 15 which is equidistant from the coordinate axes will lie only in

(1) 4^{th} quadrant (2) 1^{st} quadrant

(3*) 1^{st} and 2^{nd} quadrants (4) 1^{st}, 2^{nd} and 4^{th} quadrants

Sol. Now, $\left|\frac{15-3t}{5}\right| = |t|$

$\Rightarrow \frac{15-3t}{5} = t$ or $\frac{15-3t}{5} = -t$

$\therefore\ t = \frac{15}{8}$ or $t = \frac{-15}{2}$

So, $P\left(\frac{-15}{8}, \frac{15}{8}\right) \in 1^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in II^{st}$ Quadrant

50. The equation of a plane containing the line of intersection of the planes 2x – y – 4 = 0 and y + 2z – 4 = 0 and passing through the point (1, 1, 0) is

(1) 2x – z = 2 (2) x + 3y + z = 4 (3*) x – y – z = 0 (4) x – 3y – 2z = –2

Sol. The required plane is $(2x - y - 4) + \lambda(y + 2z - 4) = 0$ it passes through (1, 1, 0)

$\Rightarrow (2 - 1 - 4) + \lambda(1 - 4) = 0$

$\Rightarrow -3 - 3\lambda = 0 \Rightarrow \lambda = -1$

$\Rightarrow x - y - z = 0$

51. The sum of all natural numbers 'n' such that 100 < n < 200 0and H.C.F. (91, n) > 1 is

(1) 3121 (2*) 3303 (3) 3203 (4) 3221

Sol. S_A = sum of numbers between 100 and 200 which are divisible by 7.

$\Rightarrow S_A = 105 + 112 + ... + 196$

$$S_A = \frac{14}{2}[105+196] = 2107$$

S_B = Sum of numbers between 100 and 200 which are divisible by 13.

$S_B = 104 + 117 + ...+ 195 = \frac{8}{2}[104+195] = 1196$

S_C = Sum of numbers between 100 and 200 which are divisible by 7 and 13.

$S_C = 182$

$\Rightarrow$ H.C. F. (91, n) > 1 $= S_A + S_B - S_C = 3121$

52. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?

(1) $P(A | B) = P(B) - P(A)$ (2*) $P(A | B) \geq P(A)$

(3) $P(A | B) = 1$ (4) $P(A | B) \geq P(A)$

Sol. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$

(as $A \subset B \Rightarrow P(A \cap B) = P(A)$)

$\Rightarrow P(A|B) \geq P(A)$

53. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to

(1) $\frac{4}{17}$ (2) $\frac{64}{17}$ (3) $\frac{128}{17}$ (4*) $\frac{2}{17}$

Sol. $4a^2 + b^2 = 8$ …………..(1)

$$\left.\frac{dy}{dx}\right)_{(1,2)} = -\frac{4x}{y} = -2$$

$$\Rightarrow -\frac{4a}{b} = \frac{1}{2}$$

B = –8a

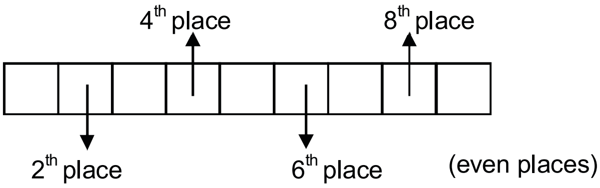
$\Rightarrow b^2 = 64a^2$

$68a^2 = 8,\ a^2 = \frac{2}{17}$

54. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is

(1) 175 (2) 160 (3) 180 (4) 162

Sol.



Number of such numbers $= |{}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{214!} = 180$

55. $\underset{x\to 0}{\text{Lim}} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals

(1) $\sqrt{2}$ (2) 4 (3) $2\sqrt{2}$ (4*) $4\sqrt{2}$

Sol. $\lim_{x\to\infty} \frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$

$= \frac{(1)^2 . (2\sqrt{2})}{\frac{1}{2}} = 4\sqrt{2}$

56. If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha,\ \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to

(1) $\frac{33}{52}$ (2*) $\frac{63}{16}$ (3) $\frac{21}{16}$ (4) $\frac{63}{52}$

Sol. $0 < \alpha+\beta = \frac{\pi}{2}$ and $\frac{-\pi}{4} < \alpha-\beta < \frac{\pi}{4}$

If cos $(\alpha+\beta) = \frac{3}{5}$ than tan $(\alpha+\beta) = \frac{4}{3}$ and if sin $(\alpha-\beta) = \frac{5}{13}$ than tan $(\alpha-\beta) = \frac{5}{12}$

(since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha) = \tan\{(\alpha+\beta) + (\alpha+\beta)\}$

$= \frac{\tan(\alpha+\beta)+\tan(\alpha+\beta)}{1-\tan(\alpha-\beta).\ \tan(\alpha-\beta)} = \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$

57. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

(1) If you are a citizen of India, then you are born in India.

(2) If you are not born in India, then you are not a citizen of India.

(3) If you are born in India, then you are not a citizen of India.

(4*) If you are not a citizen of India. then you are not born in India.

Sol. The contrapositive of a statement $p \to q$ is $\sim q \to \sim p$

Here, p : your are born in India

q : you are citizen of India

So, contrapositive of above statement is "If you are not a citizen of India, then you are not born in India".

58. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$, (x > 0) is equal to

(1*) 10 (2) 4 (3) 9 (4) 12

Sol. $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$

$\left|\sqrt{x}-2\right|+\left(\sqrt{x}\right)^2-4\sqrt{x}+2=0$

$\left|\sqrt{x}-2\right|^2+\left|\sqrt{x}-2\right|-2=0$

$\left|\sqrt{x}-2\right|=-2(\text{not possible})$ or $\left|\sqrt{x}-2\right|=1$

$\sqrt{x}-2=1,-1$

$\sqrt{x}-3,1$

X = 9, 1

Sum = 10

59. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to

(where c is a constant of integration)

(1) x + 2 sin x + 2 sin 2x + c (2) 2x + sin x + sin 2x + c

(3) 2x + sin x + 2 sin 2x + c (4*) x + 2 sin x + sin 2x + c

Sol. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx=\int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$=\int \frac{\sin 3x-\sin 2x}{\sin x}dx$

$=\int \frac{3\sin x-4\sin^3 x+2\sin x\cos x}{\sin x}dx$

$= \int (3-4\sin^2 x + 2\cos x)dx$

$= \int (3-2(1-\cos 2x) + 2\cos x)dx$

$= \int (1 + 2\cos 2x + 2\cos x)dx$

$= x + \sin 2x + 2\sin x + c$

60. If f (x) = $\frac{2-x\cos x}{2+x\cos x}$ and g(x) = $\log_e x$, (x > 0) then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is

(1) $\log_e 2$ (2) $\log_e e$ (3*) $\log_e 1$ (4) $\log_e 3$

Sol. $g(f(x)) = \ell n(f(x)) = \ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)$

$\therefore\ I = \int_0^{\pi/2}\left(\ell n\left(\frac{2-x3\cos x}{2+x.\cos x}\right) + \ell\left(\frac{2+x.\cos x}{2-x.\cos x}\right)\right)dx$

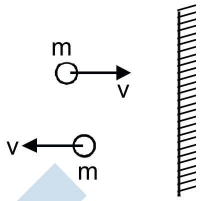
$\therefore\ I = \int_0^{\pi/2}(0)dx = 0 = \log_e(1)$

PART-C-PHYSICS

61. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of :

(1) 10^{16} N/m^2 (2) 10^4 N/m^2 (3) 10^8 N/m^2 (4*) 10^3 N/m^2

Sol. Pressure is defined as normal force per unit area.



Force is calculated as change in momentum/ time.

By this answer is 2N/m2

None of the option matches so this question must be Bonus.

Detailed solution is as following:

Magnitude of change in momentum per collision = 2mv

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{N(2mv)}{1}$$

$$= \frac{10^{22} \times 2 \times 10^{-28} \times 10^4}{1}$$

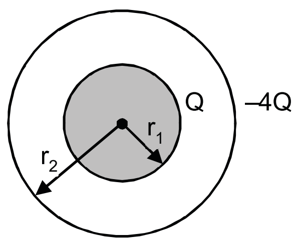
$= 2\text{N/m}^2$

62. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of -4 Q, the new potential difference between the same two surfaces is :

(1) 4V (2) –2V (3) 2 V (4*) V

Sol. As given in the first condition:

Both conducting spheres are shown.



$$V_{in} - V_{out}\left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right)$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

In the second condition:

Shell is now given charge –4Q.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

$$= \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$

Hence, we also obtain that potential difference does not depend on charge of outer sphere.

$\therefore$ P. d. remains same.

63. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be :

(1) 2 (2) 9 (3) 18 (4*) 4

Sol. Given $\frac{a_1}{a_2} = \frac{1}{3}$

Ratio of intensities, $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{9}$

Now, $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{1+3}{1-3}\right)^2 = 4$

64. A plane electromagnetic wave travels in free space along the x – direction. The electric field component of the wave at a particular point of space and time is E = 6Vm^{-1} along y-direction. Its corresponding magnetic field component, B would be :

(1) 6 × 10^{-8} T along z - direction (2*) 2 × 10^{-8} T along z - direction

(3) 2 × 10^{-8} T along y - direction (4) 6 × 10^{-8} T along x- direction

Sol. The direction of propagation of an EM wave is direction of $\vec{E} \times \vec{B}$

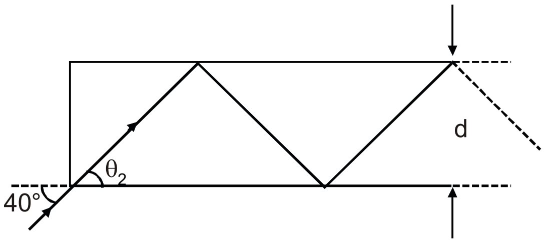
$\hat{i} = \hat{j} \times \hat{B}$

$\Rightarrow \hat{B} = \hat{k}$

$C = \frac{E}{B} \Rightarrow B = \frac{E}{C} = \frac{6}{3 \times 10^8}$

B = 2 × 10^{-8} T along z direction.

65. In figure, the optical fiber is ℓ = 2 m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40^0$, the number of reflections it makes before emerging from the other end is close to : (refractive index of fiber is 1.31 and sin 40^0 = 0.64)



(1) 45000 (2) 55000 (3) 66000 (4*) 57000

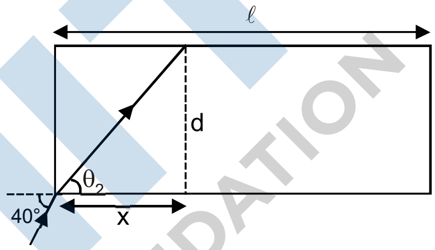
Sol. If we approximate the angle θ2 as 30° initially then answer will be closer to 57000. but if we solve thoroughly, answer will be close to 55000.

So both the answers must be awarded. Detailed solution as following.

Exact solution

By Snell's law 1.sin 40° = (1.31) sin θ_2

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$



$$\text{Now } \tan\theta_2 = \frac{64}{\sqrt{(131)^2-(64)^2}} = \frac{64}{\sqrt{13065}} \approx \frac{64}{114.3} = \frac{d}{x}$$

Now number of reflections

$$= \frac{2\times 64}{114.3\times 20\times 10^{-6}} = \frac{64\times 10^5}{114.3}$$

$\approx 55991 \approx 55000$

Approximate solution

By Snell's law 1. Sin 40° = (1.31) sin θ_2

$$\text{Sin }\theta_2 = \frac{0.64}{1.31} = \frac{64}{131} \approx 0.49$$

If assume $\Rightarrow \theta_2 \approx 30°$

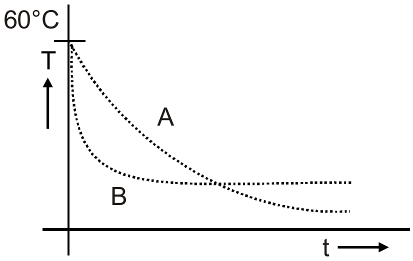
$$\text{Tan } 30° = \frac{d}{x} \Rightarrow x = \sqrt{3}d$$

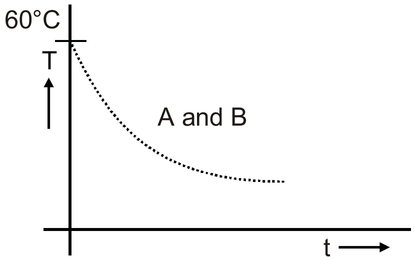
Now number of reflections

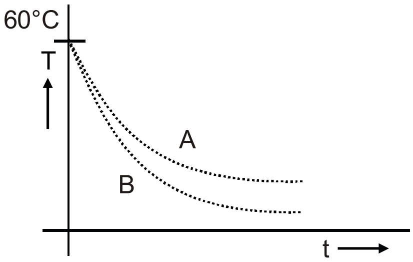
$$= \frac{\ell}{\sqrt{3}d} = \frac{2}{\sqrt{3}\times 20\times 10^{-6}} = \frac{10^5}{\sqrt{3}}$$

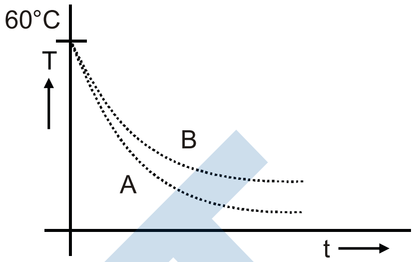
$\approx 57735 \approx 57000$

66. Two identical beakers A and B contain equal volumes of two different liquids at 60^0 C each and left to cool down. Liquid in A has density 8 × 10^2 kg/m^3 and specific heat of 2000 J $kg^{-1}K^{-1}$ while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

(1) 

(2) 

(3) 

(4*) 

Sol. $-ms\dfrac{dT}{dt} = e\sigma A\left(T^4 - T_0^4\right)$

$-\dfrac{dT}{dt} = \dfrac{e\sigma A}{ms}\left(T^4 - T_0^4\right)$; $\dfrac{dT}{dt} = \dfrac{4e\sigma AT_0^3}{ms}(\Delta T)$

$T = T_0 + (T_i - T_0)e^{-kt}$

where $k = \dfrac{4e\sigma AT_0^3}{ms}$

$k = \dfrac{4e\sigma AT_0^3}{\rho vs}$; $\left|\dfrac{dT}{dt}\right| \propto k$

$\therefore \left|\dfrac{dT}{dt}\right| \propto \dfrac{1}{\rho s}$

$\rho_A S_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$

$\rho_B S_B = 4000 \times 10^3 = 4 \times 10^6$

$\rho_A S_A < \rho_B S_B$

$\left|\dfrac{dT}{dt}\right|_A > \left|\dfrac{dT}{dt}\right|_B$

67. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is

(1) 1/2 (2) 3/5 (3) 3/2 (4*) 8/5

Sol. $M = \int_0^R \rho_0 r(2\pi r dr) = \dfrac{\rho_0 \times 2\pi \times R^5}{3}$

$$\underset{\text{(MOI about COM)}}{I_0} = \int_0^R \rho_0 r(2\pi r dr) \times r^2 = \frac{\rho_0 \times 2\pi R^5}{5}$$

By parallel axis theorem

$I = I_0 + MR^2$

$$= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15}$$

$$= MR^2 \times \frac{8}{5}$$

68. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\,\hat{i}$. The torque on the coil due to the magnetic field is :

(1) Zero (2*) $B\pi r^2 I N$ (3) $\frac{Br^2 I}{\pi N}$ (4) $\frac{B\pi r^2 I}{N}$

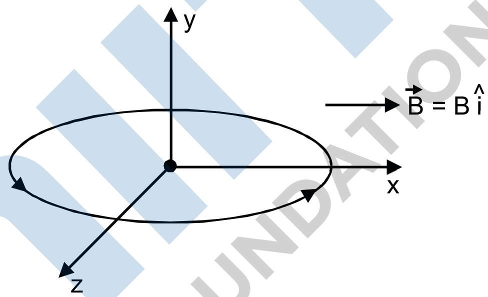
Sol. Magnetic moment of coil $= NIA\,\hat{j}$

$= NI\,(\pi r^2)\,\hat{j}$

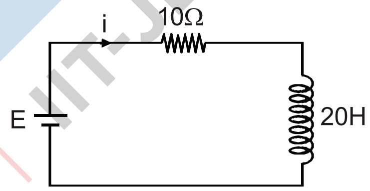
Torque on loop (coil) = $\vec{M} \times \vec{B}$

$= NI(\pi r^2)B \sin 90° (-\hat{k})$

$= NI\pi r^2 B\,(-\hat{k})$



69. A 20 Henry inductor coil is connected to a 10-ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is :



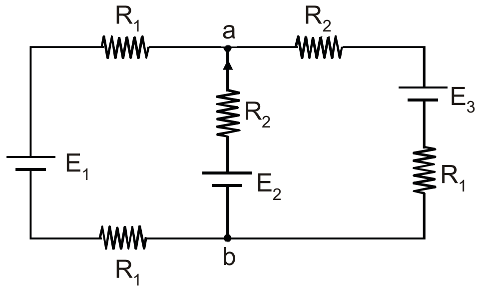
(1) $\frac{2}{\ell n2}$ (2) $\ell n2$ (2) $\frac{1}{2}\ell n2$ (4*) $2\ell n\,2$

Sol. $LIDI = \ell^2 R$

$$L \times \frac{E}{10}\left(-e^{-t/2}\right) \times \frac{-1}{2} = \frac{E}{10}\left(1 - e^{-t/2}\right) \times 10$$

$E^{-1/2} = 1 - e^{-1/2}$; $t = 2\ell n2$

70. For the circuit shown, with R_1 = 1.0 Ω, R_2 = 2.0Ω, E_1 = 2 V and $E_2 = E_3$ = 4 V, the potential difference between the point 'a' and 'b' is approximately (in V) :

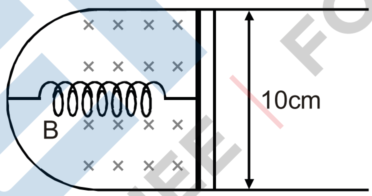


(1) 2.7 (2*) 3.3 (3) 2.3 (4) 3.7

Sol. $E_{eq} = \dfrac{\dfrac{E_1}{2R_1} + \dfrac{E_2}{R_2} + \dfrac{E_3}{2R_1}}{\dfrac{1}{2R_1} + \dfrac{1}{R_2} + \dfrac{1}{2R_1}}$

$= \dfrac{\dfrac{2}{2} + \dfrac{4}{2} + \dfrac{4}{2}}{\dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2}} = \dfrac{5}{\dfrac{3}{2}} = \dfrac{10}{3} = 3.3$

71. A thin strip 10 cm long is on a U-shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to :



(1) 10000 (2*) 5000 (3) 50000 (4) 1000

Sol. $T_0 = 2\pi\sqrt{\dfrac{m}{k}} = \dfrac{2\pi}{\sqrt{10}}$

$A = A_0 e^{-1/\gamma}$

$\therefore$ for $A = \dfrac{A_0}{e}, t = \gamma$

$t = \gamma = \dfrac{2m}{b} = \dfrac{2m}{\dfrac{B^2\ell^2}{R}} = 10^4 s$

$\therefore$ No of oscillation $\dfrac{t}{T_0} = \dfrac{10^4}{2\pi/\sqrt{10}} \approx 5000.$

72. In SI units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is :

(1*) $A^2 T^3 M^{-1} L^{-2}$ (2) $A^{-1} TML^3$ (3) $AT^{-3}ML^{3/2}$ (4) $AT^2M^{-1}L^{-1}$

Sol. Dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [MLT^{-2}A^{-2}]$

Dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$

$= [M^{-2}L^{-4}T^6A^4]^{1/2}$

$= [M^{-1}L^{-2}T^3A^2]$

73. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a celling. Given that $g = 3.1\ \pi\ ms^{-2}$, what will be the tensile stress that would be developed in the wire?

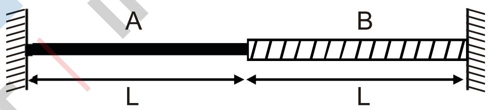
(1*) $3.1 \times 10^6\ Nm^{-2}$ (2) $4.8 \times 10^6\ Nm^{-2}$ (3) $5.2 \times 10^6\ Nm^{-2}$ (4) $6.2 \times 10^6\ Nm^{-2}$

Sol. Tensile stress in wire will be

$= \frac{\text{Tensile force}}{\text{Cross section Area}}$

$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} Nm^{-2} = 3.1 \times 10^6\ Nm^{-2}$

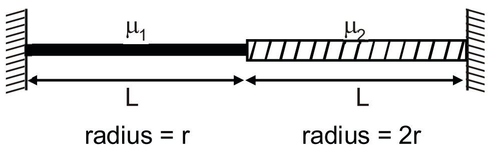
74. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is :



(1) 4 : 9 (2) 1 : 4 (3*) 1 : 2 (4) 3 : 5

Sol. Let mass per unit length of wires are μ_1 and μ_2 respectively.

$\because$ Materials are same, so density ρ is same.



$\therefore\ \therefore\ \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu$ and $\mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$

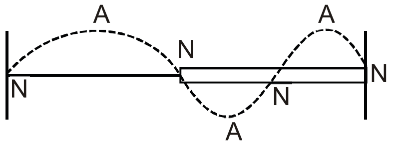
Tension in both are same = T, let speed of wave in wires are V_1 and V_2

$$V_1 = \frac{V_1}{2L} = \frac{V}{2L} \;\&\; f_{02} = \frac{v_2}{2L} = \frac{V}{4L}$$

Frequency at which both resonate is L.C.M. of

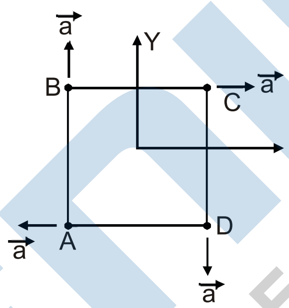
both frequencies i.e. $\frac{V}{2L}$.

Hence number of loops in wires are 1 and 2 respectively



So, ratio of number of antinodes is 1 : 2.

75. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



(1) Zero (2*) $\frac{a}{5}(\hat{i}-\hat{j})$ (3) $\frac{a}{5}(\hat{i}+\hat{j})$ (4) $a(\hat{i}+\hat{j})$

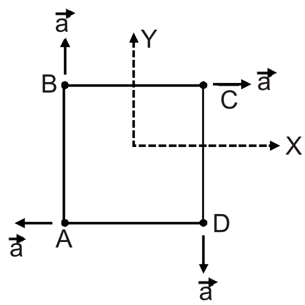
Sol. $\vec{a}_A = -a\hat{i}$; $\vec{a}_B = a\hat{j}$

$\vec{a}_C = a\hat{i}$; $\vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2m\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i}-\hat{j})$$



76. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3 , coefficient of viscosity of water = 1 mPa s)

(1) 10^2 (2) 10^6 (3*) 10^4 (4) 10^3

Sol. Reynolds number $= \frac{\rho v d}{\eta}$

Volume flow rate = × πr^2

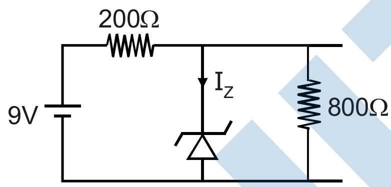
$$v = \frac{100\times10^{-3}}{60}\times\frac{1}{\pi\times25\times10^{-4}}$$

$$v = \frac{2}{3\pi} m/s$$

$$\text{Reynolds number} = \frac{10^3\times2\times10\times10^{-2}}{10^{-3}\times3\pi} = 2\times10^4$$

Order 10^4

77. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_Z through the Zener is :

(1) 7 mA (2) 17 mA (3) 15 mA (4*) 10 mA

Sol. $9 = V_z + V_{R1}$

$V_z = 5.6$ V

$V_{R1} = 9 - 5.6$

$V_{R1} = 3.4$

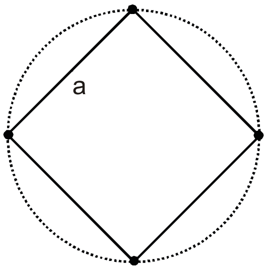
$I_{R_1} = \frac{V_{R_1}}{R} = \frac{3.4}{200}$; $I_{R_1} = 17$ mA

$V_2 = V_{R2} = I_{R2}(R_2)$

$\frac{5.6}{800} = I_{R_2}$; $I_{R_2} = 7$mA

$I_z = (17 - 7)$mA = 10 mA

78. Four identical particles of mass M are located at the corners of a square of side 'a' . What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square?

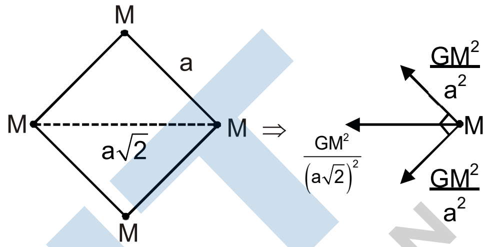


(1*) $1.16\sqrt{\frac{GM}{a}}$ (2) $1.41\sqrt{\frac{GM}{a}}$ (3) $1.35\sqrt{\frac{GM}{a}}$ (4) $1.21\sqrt{\frac{GM}{a}}$

Sol. Net force on particle towards centre of circle is

$$F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$$

$$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$



This force will act as centripetal force.

Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

$$r = \frac{a}{\sqrt{2}}, F_C = \frac{mv^2}{r}$$

$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

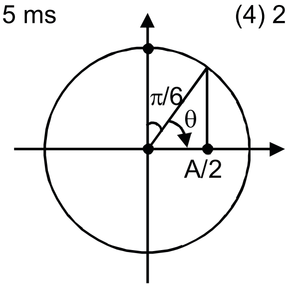
$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}} + 1\right)$$

$$v^2 = \frac{GM}{a}(1.35) \;\; ; \;\; v = 1.16\sqrt{\frac{GM}{a}}$$

79. An alternating voltage v(t) = 220 sin 100πt volt is applied to a purely resistive load of 50 Ω. Then time taken for the current to rise from half of the peak value to the peak value is :

(1) 7.2 ms (2*) 3.3 ms (3) 5 ms (4) 2.2 ms

Sol. V(t) = 220 sin (100 πt) volt time taken,



$$t = \frac{\theta}{\omega} = \frac{\frac{\pi}{3}}{100\pi} = \frac{1}{300}\text{sec}$$

= 3.3 ms

80. Radiation coming from transitions n = 2 to n= 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is :

(1) n = 2 $\rightarrow$ n = 3 (2) n = 1 $\rightarrow$ n = 4 (3*) n = 2 $\rightarrow$ n = 4 (4) n = 2 $\rightarrow$ n = 5

Sol. Energy released for tension n = 2 to n = 1 of hydrogen atom

$$E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Z = 1, N_1 = 1, n_2 = 2

$$E = 13.6 \times 1 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$E = 13.6 \times \frac{3}{4} eV$$

For He^+ ion z = 2

(A) n = 1 to n = 4

$$E = 13.6 \times 2^2 \times \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{15}{4} eV$$

(B) n = 2 to n = 5

$$E = 13.6 \times 2^2 \times \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{3}{4} eV$$

(C) n = 2 to n = 5

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 13.6 \times \frac{21}{25} eV$$

(D) n = 2 to n = 5

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{9} eV$$

81. The wavelength of the carrier waves in a modern optical fiber communication network is close to :

(1) 600 nm (2) 900 nm (3*) 1500 nm (4) 2400 nm

Sol. To minimize attenuation, wavelength of carrier waves is close to 1500 nm.

82. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V / m. The plate area is 10^{-4} m^2. What is the dielectric constant if the capacitance is 15 pF? (given ϵ_0 = 8.86 × 10^{-12} C^2 / Nm^2)

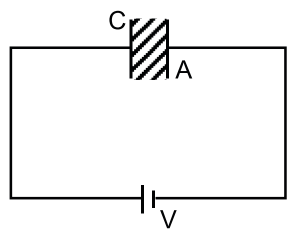
(1) 3.8 (2) 6.2 (3) 4.5 (4*) 8.5

Sol. A = 10^{-4} m^2

E_{max} 10^6 V/m

C = 15 μF

$$C = \frac{k\varepsilon_0 A}{d} \quad ; \quad \frac{Cd}{\varepsilon_0 A} = k$$



$$k = \frac{15\times10^{-12}\times500\times10^{-6}}{8.86\times10^{-12}\times10^{4}} = \frac{15\times5}{8.86} = 8.465$$

$k \approx 8.5$

83. An upright object is placed at a distance of 40 cm in front a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be :

(1) 20 cm from the convergent mirror, twice the size of the object

(2) 40 cm from the convergent lens, twice the size of the object

(3) 40 cm from the convergent mirror, same size as the object

(4*) 20 cm from the convergent mirror, same size as the object

Sol. There will be 3 phenomenon

(i) Refraction from lens

(ii) Reflection from mirror

(iii) Refraction from lens

After these phenomena. Image will be on object and will have same size.

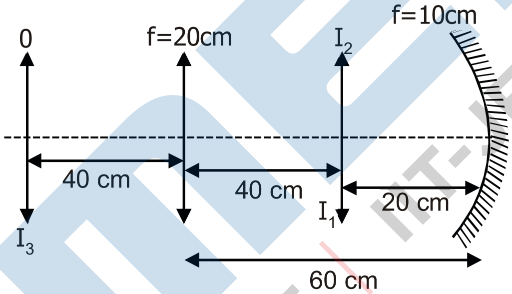
None of the option depicts so this question is Bonus.

1^{st} refraction u = –40 cm ; f = +20 cm

$\Rightarrow$ v = +40 cm (image I_1) and m_1 = –1

for reflection

u = –20 cm ; f = –10 cm



$\Rightarrow$ v = –20 cm (image I_2) and m_2 = –1

2^{nd} refraction

u = –40 cm ; f = +20 cm

$\Rightarrow$ v = +40 cm (image I_3) and m_3 = –1

Total magnification = $m_1 \times m_2 \times m_3$ = –1 and final image is formed at distance 40 cm from convergent lens and is of same size as the object.

84. Ship A is sailing towards north - east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km / hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in :

(1) 3.2 hrs (2) 4.2 hrs (3) 2.2 hrs (4*) 2.6 hrs

Sol. If we take the position of ship 'A' as origin then position and velocities of both ships can be given as:

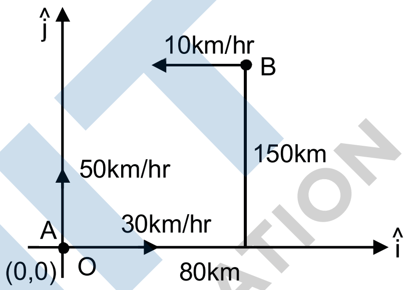
$\vec{v}_A = (30\hat{i} + 50\hat{j})$ km / hr

$\vec{v}_B = -10\hat{i}$ km / hr ; $\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = -(80\hat{i} + 150\hat{j})$ km

Time after which distance between them will be minimum

$t = -\frac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2}$;



Where $\vec{r}_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{V}_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j})(-40\hat{i} - 50\hat{j})$ km / hr

$$\therefore t = \frac{(80\hat{i} + 150\hat{j}) \cdot (-40\hat{i} - 50\hat{h})}{|-40\hat{i} - 50\hat{h}^2|}$$

$$= \frac{3200 + 7500}{4100} \text{hr} = \frac{10700}{4100} \text{hr} = 2.6\text{hrs}$$

85. Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ_1 of the final particle, is given by :

(1) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ (2*) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$ (3) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (4) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

Sol.  $\frac{h}{\lambda_1} = P_1$  $P_2 = \frac{h}{\lambda_2}$

$\vec{P}_1 = \frac{h}{\lambda_1}\hat{i}$ and $\vec{P}_2 = \frac{h}{\lambda_2}\hat{j}$

Using momentum conservation

$\vec{P} = \vec{P}_1 + \vec{P}_2 = \frac{h}{\lambda_1}\hat{i} + \frac{h}{\lambda_2}\hat{j}$

$|\vec{P}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

86. A 200Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be :

(1) 400 Ω (2) 300 Ω (3) 100 Ω (4*) 500 Ω

Sol. When red is replace with green 1st digit changes to 5 so new resistance will be 500 Ω.

87. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to :

(1) 10^8 Nm^{-2} (2) 10^4 Nm^{-2} (3*) 10^6 Nm^{-2} (4) 10^3 Nm^{-2}

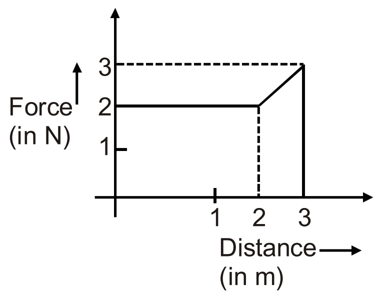
Sol. Energy of catapult $= \frac{1}{2} \times \left(\frac{\Delta\ell}{\ell}\right)^2 \times Y \times A \times \ell$

= Kinetic energy of the ball $= \frac{1}{2}mv^2$

Thererfore, $\frac{1}{2} \times \left(\frac{20}{42}\right)^2 \times Y \times \pi \times 3^2 \times 10^{-6} \times 42 \times 10^{-2} = \frac{1}{2} \times 2 \times 10^{-2} \times (20)^2$

$Y = 3 \times 10^6$ Nm^2

88. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is:



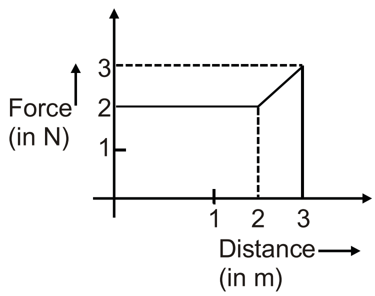
(1) 2.5 J (2) 4 J

(3*) 6.5 J (4) 5 J

Sol. According to work energy theorem.

Work done by force on the particle = Change in KE

Work done = Area under F-x graph $= \int F \cdot dx = 2 \times 2 + \frac{(2+3)\times 1}{2}$



$W = KE_{final} - KE_{initial} = 6.5$

$KE_{initial} = 0$

$KE_{final} = 6.5$ J

89. A thermally insulated vessel contains 150g of water at 0^0C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0^0C itself. The mass of evaporates water will be closest to : (Latent heat of vaporization of water = 2.10×10^6 J kg^{-1} and latent heat of fusion of water = 3.36×10^5 J kg^{-1})

(1) 130 g (2) 35 g (3*) 20 g (4) 150 g

Sol. Suppose 'm' gram of water evaporates then, heat required

$\Delta Q_{req} = mL_v$

Mass that converts into ice = (150 – m)

So, heat released in this process

$\Delta Q_{rel} = (150 - m) L_f$

Now,

$\Delta Q_{rel} = \Delta Q_{req}$

$(150 - m) L_f = mL_v$

$M(L_f + L_v) = 150 L_f$

$m = \frac{150L}{L_f + L_v}$; $m = 20g$

90. The bob of a simple pendulum has mass 2 g and a charge of 5.0 μC. It is at rest in a unifrom horizontal electric field of intensity 2000 V / m. At equilibrium, the angle that the pendulum makes with the vertical is (take g = $10 m/s^2$)

(1) $\tan^{-1}(5.0)$ (2*) $\tan^{-1}(0.5)$ (3) $\tan^{-1}(2.0)$ (4) $\tan^{-1}(0.2)$

Sol. $\tan\theta = \frac{qE}{mg} = \frac{5 \times 10^{-6} \times 2000}{2 \times 10^{-3} \times 10}$

$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$